OCR
RECOGNISING ACHIEVEMENT

Thursday 21 June 2012 – Afternoon

A2 GCE MATHEMATICS

4730 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

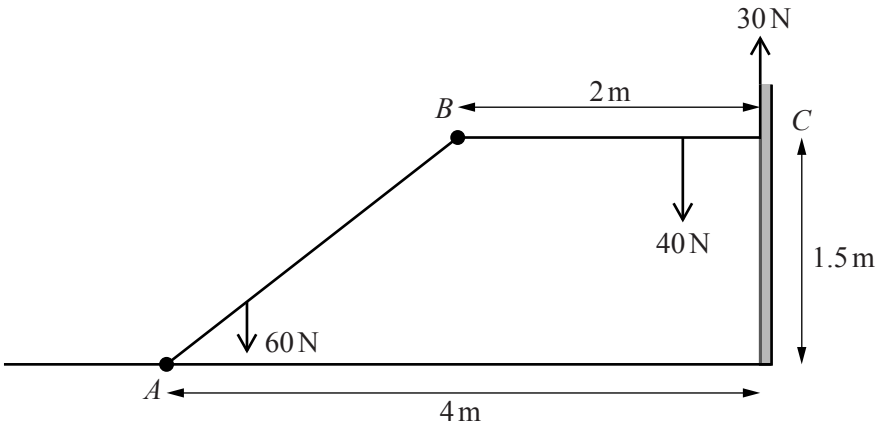
INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1



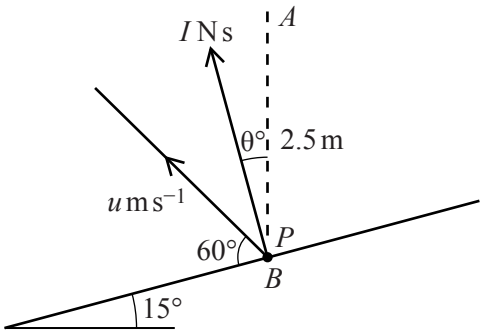
Two non-uniform rods AB and BC have weights 60 N and 40 N respectively. The rods are freely jointed to each other at B. The rod AB is freely jointed to a fixed point on horizontal ground at A and the rod BC rests against a vertical wall at C. The rod BC, whose length is 2 m, is horizontal at a height of 1.5 m above the ground. The point A is 4 m from the wall. The frictional force exerted on BC at C has magnitude 30 N (see diagram). The coefficient of friction between the rod BC and the wall is 0.75.

(i) Find the distance of the centre of mass of BC from B. **[2]**

(ii) Given that the rod BC is on the point of slipping downwards at C, find the magnitude and direction of both the vertical component and the horizontal component of the force exerted on AB at B. **[4]**

(iii) Find the distance of the centre of mass of AB from A. **[3]**

2



B is a point on a smooth plane surface inclined at an angle of 15° to the horizontal. A particle P of mass 0.45 kg is released from rest at the point A which is 2.5 m vertically above B. The particle P rebounds from the surface at an angle of 60° to the line of greatest slope through B, with a speed of u m s^{-1}. The impulse exerted on P by the surface has magnitude I N s and is in a direction making an angle of θ° with the upward vertical through B (see diagram).

(i) Explain why $\theta = 15$. **[1]**

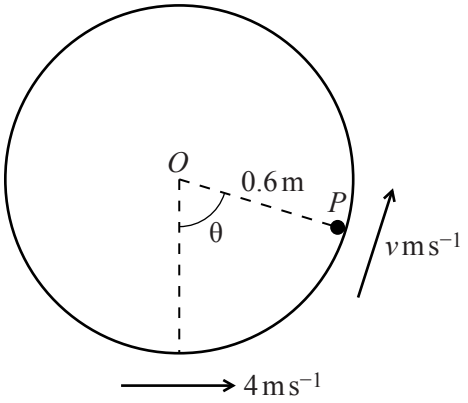
(ii) Find the values of u and I. **[7]**

3 A particle P of mass m kg is released from rest and falls vertically. When P has fallen a distance of x m it has a speed of v m s^{-1}. The only forces acting on P are its weight and air resistance of magnitude $\frac{1}{400}mv^2$ N.

(i) Find v^2 in terms of x and show that v^2 must be less than 3920. **[8]**

(ii) Find the speed of P when it has fallen 100 m. **[2]**

4



A hollow cylinder is fixed with its axis horizontal. The inner surface of the cylinder is smooth and has radius 0.6 m. A particle P of mass 0.45 kg is projected horizontally with speed $4\,\text{m s}^{-1}$ from the lowest point of a vertical cross-section of the cylinder and moves in the plane of the cross-section, which is perpendicular to the axis of the cylinder. While P remains in contact with the surface, its speed is $v\,\text{m s}^{-1}$ when OP makes an angle θ with the downward vertical at O, where O is the centre of the cross-section (see diagram). The force exerted on P by the surface is R N.

(i) Show that $v^2 = 4.24 + 11.76\cos\theta$ and find an expression for R in terms of θ. **[6]**

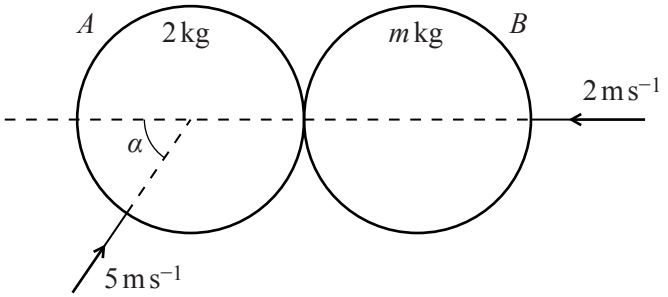
(ii) Find the speed of P at the instant when it leaves the surface. **[4]**

5 One end of a light elastic string, of natural length 0.78 m and modulus of elasticity $0.8mg$ N, is attached to a fixed point O on a smooth plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{5}{13}$. A particle P of mass m kg is attached to the other end of the string. P is released from rest at O and moves down the plane without reaching the bottom. Find

(i) the maximum speed of P in the subsequent motion, **[6]**

(ii) the distance of P from O when it is at its lowest point. **[4]**

6



Two smooth uniform spheres A and B, of equal radius, have masses 2 kg and m kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision, A has speed $5\,\text{m s}^{-1}$ and is moving towards B at an angle of α to the line of centres, where $\cos\alpha = 0.6$. B has speed $2\,\text{m s}^{-1}$ and is moving towards A along the line of centres (see diagram). As a result of the collision, A's loss of kinetic energy is 7.56 J, B's direction of motion is reversed and B's speed after the collision is $0.8\,\text{m s}^{-1}$. Find

(i) the speed of A after the collision, **[3]**

(ii) the component of A's velocity after the collision, parallel to the line of centres, stating with a reason whether its direction is to the left or to the right, **[3]**

(iii) the value of m, **[3]**

(iv) the coefficient of restitution between A and B. **[2]**

7 S_A and S_B are light elastic strings. S_A has natural length 2 m and modulus of elasticity 120 N; S_B has natural length 3 m and modulus of elasticity 180 N. A particle P of mass 0.8 kg is attached to one end of each of the strings. The other ends of S_A and S_B are attached to fixed points A and B respectively, on a smooth horizontal table. The distance AB is 6 m. P is released from rest at the point of the line segment AB which is 2.9 m from A.

(i) For the subsequent motion, show that the total elastic potential energy of the strings is the same when $AP = 2.1$ m and when $AP = 2.9$ m. Deduce that neither string becomes slack. **[3]**

(ii) Find, in terms of x, an expression for the acceleration of P in the direction of AB when $AP = (2.5 + x)$ m. **[3]**

(iii) State, giving a reason, the type of motion of P and find the time taken between successive occasions when P is instantaneously at rest. **[3]**

For the instant 0.6 seconds after P is released, find

(iv) the distance travelled by P, **[3]**

(v) the speed of P. **[2]**

Question			Answer	Marks	Guidance
1		(i)	[$40d = 30 \times 2$] Distance is 1.5 m	M1 A1 **[2]**	For taking moments about B for BC
		(ii)	$30 = 0.75\ R$ Horizontal component on AB at B is 40 N to the left For resolving forces on BC vertically, or taking moments about C Vertical component on AB at B is 10 N down	B1 B1 M1 A1 **[4]**	 $Y + 30 = 40$, or $40 \times ½ = Y \times 2$ Accept directions on diagram, if not contradicted in text SR A1 if both magnitudes correct but directions wrong/not stated
		(iii)	$(+/-)10 \times 2 + 60 \times 0.8d = (+/-)40 \times 1.5$ Distance is 0.833 m	M1 A1 FT A1 **[3]**	For taking moments about A for AB FT magnitudes of components at B; need to use '$x = d\cos\theta$' May see moments about A for ABC ($60 \times 0.8d + 40 \times 3.5 = 30 \times 4 +$ '40' $\times 1.5$) or moments about B for AB – need to get equation with only 'd' unknown for M1
2		(i)	Since plane is smooth impulse is perpendicular to plane(so $\theta = 15$)	B1 **[1]**	
		(ii)	Use of $v^2 = (u^2) + 2 \times g \times 2.5$ $v = 7$ ms^{-1} after impact: Speed parallel to plane is 7sin15° $u = 7\sin15° / \cos 60°$ $u = 3.62$ $I = 0.45(7\cos15^o + u\sin60°)$ $I = 4.45$ Or For using a triangle with sides 3.15 (0.45 x 7), I and 0.45 x u (or 7, I/0.45 and u) and correct angles 135°, 15° and 30° Use of sin rule or cos rule (correct) $u = 3.62$ $I = 4.45$	M1 A1 B1 M1 A1 M1 A1 **[7]** M1 A1 M1 A1 A1	 1.81(173…) Allow sin/cos errors Allow sin/cos errors or $I = 0.45(7\cos15^o + 7\sin15°\tan60°)$ 4.45477…. May see $e = 0.464$ Need 2 correct sides and 1 correct angle All correct OR $I\cos15° = 3.15 + 0.45\ u\cos45°$ M1 $I\sin15° = mu\cos45°$ B1 Solve sim equations M1, dep attempt at two comps of I Answers A1A1

Question			Answer	Marks	Guidance
3		(i)		M1	For using N's 2nd law with $a = v\,\mathrm{d}v/\mathrm{d}x$; 3 terms
			$v\,\mathrm{d}v/\mathrm{d}x = g - 0.0025v^2$	A1	
			$\int \frac{v\,dv}{g - 0.0025v^2} = \int dx$	M1	For correctly separating variable and attempting to integrate
			$-200\ln(g - 0.0025v^2) = x\ (+A)$	A1	
			$A = -200\ln g$	M1*	Attempt to find A from $B\ln(C - Dv^2)$
			$[g - 0.0025v^2 = ge^{-0.005x}]$	*M1	For transposing equation to remove ln
			$v^2 = 400g(1 - e^{-0.005x})$	A1	
			$0 < e^{-0.005x} \leq 1 \rightarrow v^2$ cannot reach $400g$ ie cannot reach 3920	B1	dependent on getting other 7 marks. Need '0 <' oe
				[8]	
		(ii)	$v^2 = 400g(1 - e^{-0.5})$	M1	For substituting for x and evaluating v must have $v^2 = A + Be^{Cx}$ for (i), but not neces in this form
			Speed of P is 39.3 ms^{-1}	A1	
				[2]	
4		(i)		M1	For using the pce condone sin/cos and sign errors; need KE before and after and difference in PE
			$\frac{1}{2}mv^2 + mg(0.6)(1 - \cos\theta) = \frac{1}{2}m4^2$	A1	
			$v^2 = 4.24 + 11.76\cos\theta$	A1	AG
				M1	For using Newton's 2nd law, condone sin/cos and sign erorrs; 3 terms needed
			$R - 0.45g\cos\theta = 0.45v^2/0.6$	A1	
			$R = 3.18 + 13.23\cos\theta$	A1	
				[6]	
		(ii)		M1	For using $R = 0$
			$\cos\theta = -3.18/13.23$	A1 FT	$-0.24036\ldots$ or $-106/441$ or $\theta = 103.9°$ ft from $R = A + B\cos\theta$, where $A, B \neq 0$
			$[v^2 = 4.24 - 11.76 \times 3.18/13.23]$	M1	For substituting for $\cos\theta$
			Speed is 1.19 ms^{-1}	A1	CAO without wrong working
				[4]	

Question			Answer	Marks	Guidance
5		(i)	$[0.8mgx/0.78 = mg(5/13)]$	M1	For resolving forces and using $T = \lambda x / L$ at equilibrium position
			$x = 0.375$	A1	Accept 1.155 for $e + l$
			PE $= mg(0.78 + 0.375) \times 5/13$	B1 FT	FT value of x
			EE $= 0.8mg \times 0.375^2 \div (2 \times 0.78)$	B1 FT	FT value of x
			$[\frac{1}{2} mv^2 = m(4.353... - 0.7067...)]$	M1	For using $\frac{1}{2} mv^2$ = PE loss – EE gain
			Maximum speed is 2.70 ms^{-1}	A1	
				[6]	
			OR at extension x		
			PE $= mg(x + 0.78) \times \frac{5}{13}$	B1	
			EE $= \dfrac{0.8mgx^2}{2 \times 0.78}$	B1	
			$mg(x + 0.78) \times \dfrac{5}{13} = \dfrac{1}{2} mv^2 + \dfrac{0.8mgx^2}{2 \times 0.78}$	M1	For using $\frac{1}{2} mv^2$ = PE loss – EE gain
			$v^2 = -10.05x^2 + 7.53x + 5.88$		$v^2 = -\dfrac{40 \times 9.8}{39} x^2 + \dfrac{98}{13} x + \dfrac{9.8 \times 3.9 \times 2}{13}$
			$v^2 = -10.05(x^2 - 0.749x - 0.585)$		$v^2 = -\dfrac{392}{39}(x^2 - \dfrac{3}{4} x - \dfrac{3 \times 3.9 \times 2}{40})$
			for attempting to complete square	M1	
			$v^2 = -10.05((x - 0.375)^2 - 0.726)$	A1	$v^2 = -\dfrac{392}{39}((x - \dfrac{3}{8})^2 - 0.725625)$
			Max speed is 2.70 ms^{-1}	A1	
					Note, after getting equation for v^2, can instead Differentiate v^2 wrt x M1 Establish max at $x = 0.375$ A1 Max speed 2.70 ms^{-1} A1

Question			Answer	Marks	Guidance
		(ii)		M1*	For using PE loss = EE gain
			$mg(0.78 + x) \times 5/13 = 0.8mgx^2 \div (2 \times 0.78)$	A1	or $mg(x) \times 5/13 = 0.8mg(x - 0.78)^2 \div (2 \times 0.78)$ if $PO = x$ or $mg(x+0.78+0.375) \times 5/13 = 0.8mg(x + 0.375)^2 \div (2 \times 0.78)$ if $PO = x + 0.78 + 0.375$
			$[x^2 - 0.75x - 0.585 = 0$ if x is extension]	*M1	For arranging in quadratic form and attempting to solve All nec terms required
			$x = 1.2268$ so Distance is 2.01 m	A1	$[x^2 - 2.31x + 0.6084 = 0$ if $PO = x]$ $[20x^2 = 14.5125$, if $PO = x + 0.78 + 0.375]$
				[4]	$[x = 2.0068]$ $[x = 0.8518\ldots]$
			OR put v = 0 in v^2 equation from above	M1A1ft	
			Solve to get $x = 1.23$ (+0.78) = 2.01 m	M1A1	

Question			Answer	Marks	Guidance
6		(i)		M1	For using ½ $m(u^2 - v^2) = 7.56$ and solving for v; *must use '5', allow sign error/ missing ½, missing m.*
			½ × $2(5^2 - v^2) = 7.56$ ($v^2 = 17.44$)	A1	
			Speed is 4.18 ms^{-1}	A1	Do not award if this is not candidate's final answer.
				[3]	
		(ii)	$v_{Ay} = u_{Ay} = 5\sin\alpha = 4$	B1	
			$[v_{Ax}^2 + 4^2 = 17.44 \rightarrow v_{Ax}^2 = 1.44]$	M1	For using $v_{Ax}^2 + v_{Ay}^2 = 17.44$
			$v_{Ax} = \pm 1.2$ and v_{Ax} must be less than 0.8		
			➔Component has magnitude 1.2 ms^{-1} and direction to the left	A1	
				[3]	
		(iii)		M1	For using the pcm parallel to loc must use 5cosα, 2, 0.8 and '1.2', 4 terms or equivalent, allow sign errors, condone one mass missing
			$2 \times 3 - m \times 2 = 2 \times (-1.2) + m \times 0.8$	A1 FT	FT incorrect v_{AX}
			$m = 3$	A1	CAO
				[3]	
		(iv)	$[e(3 + 2) = (1.2 + 0.8)]$	M1	For using NEL with their '1.2' and 5cosα, 2 and 0.8; allow sign errors. Must be right way up
			$e = 0.4$	A1	
				[2]	

Question			Answer	Marks	Guidance
7		(i)	$E_{(AP=2.9)} = 120 \times 0.9^2/4 + 180 \times 0.1^2/6$ $= (24.3 + 0.3)$ and $E_{(AP=2.1)} = 120 \times 0.1^2/4 + 180 \times 0.9^2/6$ $= (0.3 + 24.3)$ ➔same for each position Conservation of energy ➔ $v = 0$ when AP = 2.1, string taut here so taut throughout motion – oe,	M1 A1 B1 **[3]**	For using EPE = $\lambda x^2/2L$ for both strings for one position 24.6 seen twice Need to point out that $v = 0$ when $AP = 2.1$ or KE = 0 Dep on M1A1
		(ii)	$T_A = 120(0.5 + x)/2$, $T_B = 180(0.5 – x)/3$ $[(30 – 60x) – (30 + 60x) = (+/–)0.8a]$ $a = –150x$	B1 M1 A1 **[3]**	soi For using Newton's 2nd law; allow omission of 0.8 With no wrong working
		(iii)	SHM because $a = –k$ (where $k > 0$) $[T = 2\pi / \sqrt{150}\ ]$ Time interval is 0.257 s	M1 M1 A1 FT **[3]**	SHM because $a = – \omega^2 x$ or in words For using $T = 2\pi /n;$ must follow from (ii) FT $\pi \div$ candidate's n 0.256509…
		(iv)	$[x = 0.4 \cos(\sqrt{150} \times 0.6) = 0.194\ ]$ [distance = $4a + (a – 0.194)$] Distance travelled is 1.81 m	M1 M1 A1 **[3]**	For using $x = a\cos(0.6n)$, where n follows from (ii) and a is numerical. For using $T < 0.6 < 1.25\ T$ ➔ distance = $4\ a + (\ a – x)$; may be implied by $1.6 <$ distance < 2.0 CAO, no wrong working
		(v)	 Speed is 4.29 ms^{-1}.	M1 A1 **[2]**	For using $\dot{x} = -an\sin(0.6n)$, where n follows from (ii) Or using $v^2 = n^2(a^2 – x^2)$, where n follows from (ii) and x follows from (iv) or using $\dot{x} = an\cos(0.6n)$ if $x = a\sin(0.6n)$ used in (iv), where n follows from (ii) Condone −4.29